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AUCTIONING INCENTIVE CONTRACTS
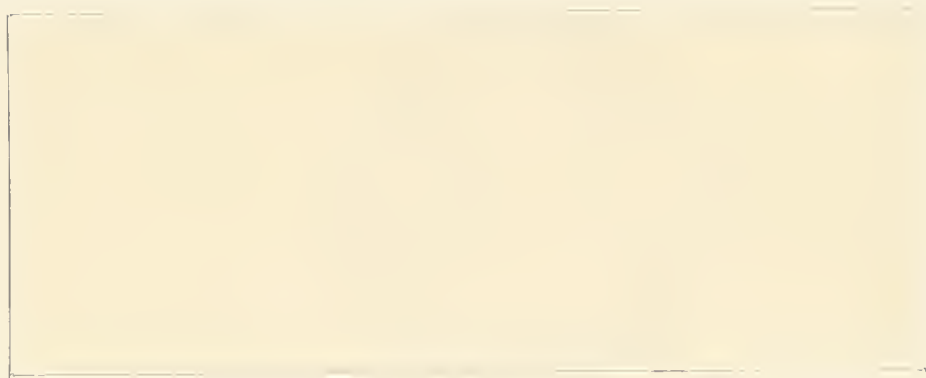
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Abstract

This paper draws a remarkably simple bridge between auction
theory and incentive theory. It considers the auctioning of an
indivisible project between several firms. The firms have priva-
te information about their future cost at the bidding stage, and
the selected firm ex-post invests in cost reduction. We show
that :

1) The optimal auction can be implemented by a dominant strategy
auction which uses both information about the first bid and the
second bid.

2) The winner faces a (linear) incentive contract.

3) The fixed transfer to the winner decreases with his announ-
ced expected cost and increases with the second lowest announced
expected cost.

4) The share of cost overruns born by the winner decreases
with the winner's announced expected cost.

1 - Introduction

In Laffont and Tirole (1984) we studied the optimal contract between a Government and a firm realizing a project for the Government. In our model both the firm and the Government are risk neutral, but the Government faces a double asymmetry of information, not knowing the exact productivity parameter of the firm and not observing its effort level. With socially costly transfers the optimal strategy for an utilitarian Government is to offer a menu of so-called incentive contracts each composed of a fixed payment, function of the announced cost, and of a linear sharing of overruns, i.e. of the difference between ex-post observed costs and announced costs. The optimal contract trades off the truthfull elicitation of information about productivity (which would lead to a cost plus contract) and the ex-post inducement of an appropriate level of effort (which would lead to a fixed price contract).

When several firms are possible candidates to realize the project, it has been argued by Demsetz (1968) that competitive bids should be elicited. In this paper we extend our analysis to the case of multiple firms. However, the model is meant to formalize a one shot project and we leave aside the questions raised by the auctioning of an activity repeated over time [1] (see Williamson (1976)).

In section 2 we set up the model ; n firms can realize a project which has a fixed large value for the Government. Under perfect information, the Government would select the most efficient firm and impose an optimal level of effort independently of the efficiency level of the firm. Section 3 characterizes the optimal Bayesian auction for an utilitarian Government. Under some assumptions which exclude bunching and random auctions, the best auction awards the project to the firm which announces the smallest expected cost. The type of contract which associates the Government with the selected firm is then similar to the one

derived in the case of a single firm. It can be written as the sum of a fixed payment function of the announced cost and of a linear sharing of overruns ; the coefficient characterizing this sharing rule is a function of the announced cost. This dichotomy property is the main result of this paper which can then rely on Laffont and Tirole (1984) for a detailed study of the optimal contract. Section 4 constructs a dominant strategy-auction which implements the optimum and section 5 concludes.

2 - The model

We assume that n firms can participate in the auction. Each firm i, i = 1,...,n, is able to realize the indivisible project for a cost :

(1) $$c^i = \beta^i - e^i$$

where e^i is manager i's ex-post effort level and β^i is firm i's efficiency parameter.

The efficiency parameters (β^i) are drawn independently from the same distribution with a cumulative distribution function F(.) on the interval $[\underline{\beta}, \bar{\beta}]$ and a differentiable density function f which is bounded below by a strictly positive number on $[\underline{\beta}, \bar{\beta}]$. Moreover, F(.) is common knowledge and satisfies the monotone hazard rate property.

Assumption 1 : $\frac{F}{f}$ is non decreasing

A1 facilitates the analysis by preventing bunching in the one firm problem [2].

Manager i, i = 1,...,n, has the utility function :

(2) $$t^i - \psi(e^i)$$

where t^i is the net (i.e. in addition to cost) monetary transfer that he receives from the Government and $\psi(e^i)$ is his disutility of effort, $\psi' > 0$, $\psi'' > 0$. Moreover, we postulate :

Assumption 2 : $\psi''' \geq 0$

A2 facilitates the analysis by ensuring that in the one firm problem there is no need for using stochastic mechanisms (and similarly in the n-firm case, see Appendix 1).

Let S be the social utility of the project. Under perfect information the Government selects the firm with the lowest parameter β, say firm i, and makes a transfer only to that firm. The gross payment made by the Government to firm i is $t^i + C^i$. The social cost of one unit of money is $1+\lambda$, $\lambda > 0$, so that the social net utility of the project for an utilitarian Government is :

(3)
$$S-(1+\lambda)(t^i+C^i)+t^i-\psi(e^i) =$$
$$S-\lambda U^i-(1+\lambda)(C^i+\psi(e^i))$$

where U^i denotes firm i's utility level.

Procurement in which the principal is a private firm would lead to a different objective function of the type, profits minus transfers. None of our results would be affected ; the important feature is that the principal dislikes transfers.

Each firm has outside opportunities (its individual rationality level) normalized to zero. Under perfect information, the optimal level of effort of the firm selected should be determined by $\psi'(e^{*i}) = 1$ (marginal disutility of effort equal marginal cost savings), and the net transfer to that firm should be $\psi(e^{*i})$.

However, the Government observes neither (β^i) nor (e^i). Ex-post he observes the realized cost of the firm which has been chosen to carry out the project. To select a firm and regulate it, he organizes an auction as explained below.

3 - The optimal Bayesian auction

Firms bid simultaneously by announcing efficiency parameters [3] $(\tilde{\beta}^1,\ldots,\tilde{\beta}^n) = \tilde{\beta}$. Let $x^i(\tilde{\beta})$ be the probability that firm i is selected to carry out the project. We must have :

(4)
$$\sum_{i=1}^{n} x^i(\tilde{\beta}) \leq 1 \qquad \text{for any } \tilde{\beta}$$

(5)
$$x^i(\tilde{\beta}) \geq 0 \qquad i = 1,\ldots,n \qquad \text{for any } \tilde{\beta}$$

Since both the Government and managers are risk neutral there is no need to consider random transfers. Let $t^i(\tilde{\beta})$ be manager i's expected transfer as a function of the announced bids. Ex-ante manager i's expected utility is :

$$(6) \qquad \underset{\tilde{\beta}^{-i}}{E} \{t^i(\tilde{\beta}) - x^i(\tilde{\beta})\psi(e^i)\}$$

where $\tilde{\beta}^{-i} = (\tilde{\beta}^1,\ldots, \tilde{\beta}^{i-1}, \tilde{\beta}^{i+1},\ldots, \tilde{\beta}^n)$.

The ex-post observability of cost enables us to rewrite (6) as :

$$(7) \qquad \underset{\tilde{\beta}^{-i}}{E} \{t^i(\tilde{\beta}) - x^i(\tilde{\beta})\psi(\beta^i-C^i(\tilde{\beta}))\}$$

where $C^i(\tilde{\beta})$ is the cost level that the Government requires the firm to reach given announcements $\tilde{\beta}$.

We look for mechanisms $(x^i(\tilde{\beta}),C^i(\tilde{\beta}),t^i(\tilde{\beta}))$ which induce a truthtelling Bayesian Nash equilibrium. From appendix 2, a necessary condition for truthtelling is :

$$(8) \quad \frac{\partial}{\partial\beta^i} \underset{\beta^{-i}}{E} t^i(\beta) = \frac{\partial}{\partial\beta^i} \underset{\beta^{-i}}{E} [x^i(\beta)\psi(\beta^i-C^i(\beta))]$$

$$dF \ a.e$$
$$for \ i = 1,\ldots,n$$

(Differentiability of these expectations almost everywhere results from their monotonicity which in turn stems from incentive compatibility).

If moreover the following condition (9) is satisfied (8) is sufficient :

$$(9) \qquad \frac{\partial C^i}{\partial\beta^i} \geq 0 \ . \qquad\qquad \frac{\partial}{\partial\beta^i} \underset{\beta^{-i}}{E} x^i(\beta) \leq 0$$

In the sequel we first ignore the second order conditions (9) and check later that they are indeed satisfied at the optimum.

Let $U^i(\beta^i)$ be manager i's expected utility level when telling the truth :

(10)
$$U^i(\beta^i) = \underset{\beta^{-i}}{E}\ [t^i(\beta) - x^i(\beta)\psi(\beta^i - C^i(\beta^i))]$$

From (7) and (8) we have :

(11)
$$\dot{U}^i(\beta^i) = -\underset{\beta^{-i}}{E}\ [x^i(\beta)\psi'(\beta^i - C^i(\beta))]$$

From (10) we see that U^i is non increasing in β^i so that manager i's individual rationality level is satisfied if it is satisfied at $\beta^i = \bar{\beta}$. As the Government's objective function is decreasing in U^i, it will be tight at $\bar{\beta}$, i.e.

(12)
$$U^i(\bar{\beta}) = 0 \qquad\qquad i = 1,\ldots,n$$

Suppose for simplicity that S is so large that even a firm with type $\bar{\beta}$ is worth selecting.

The maximand of an utilitarian Government is :

(13)
$$\left(\sum_{i=1}^{n} x^i\right) S - (1+\lambda) \sum_{i=1}^{n} t^i - (1+\lambda) \sum_{i=1}^{n} x^i C^i + \sum_{i=1}^{n} (t^i - x^i \psi^i) =$$

$$\left(\sum_{i=1}^{n} x^i\right) S - \lambda \sum_{i=1}^{n} U^i - (1+\lambda) \sum_{i=1}^{n} x^i (C^i + \psi^i)$$

The Government's optimization problem under incomplete information is then :

(14)
$$\underset{(x^i(.),C^i(.),U^i(.))}{\text{Max}} \underset{\beta}{E} \left[\left(\sum_{i=1}^{n} x^i(\beta)\right) S - \lambda \sum_{i=1}^{n} U^i(\beta^i) \right.$$

$$\left. -(1+\lambda) \sum_{i=1}^{n} x^i(\beta)[C^i(\beta) + \psi(\beta^i - C^i(\beta))] \right]$$

S.T.

$$(15) \quad \dot{U}^i(\beta^i) = - \underset{\beta^{-i}}{E} \{ x^i(\beta) \psi'(\beta^i - c^i(\beta)) \} \qquad dF \text{ a.e} \qquad i = 1, \ldots, n$$

$$(16) \quad U^i(\bar{\beta}) = 0 \qquad\qquad\qquad\qquad i = 1, \ldots, r$$

$$(17) \quad - \sum_{i=1}^{n} x^i(\beta) + 1 \geq 0 \text{ for any } \beta$$

$$(18) \quad x^i(\beta) \geq 0 \quad \text{ for any } \beta \qquad\qquad i = 1, \ldots, n$$

Without loss of generality we can assume that $\psi(0) = 0$ and $\psi'(0) = 0$ (otherwise in the next proofs write

$$\psi(x) = \psi(0) + \tilde{\psi}(x) \qquad \text{with } \tilde{\psi}(0) = 0$$
$$\psi'(x) = \psi'(0) + \tilde{\tilde{\psi}}'(x) \text{ with } \tilde{\tilde{\psi}}'(0) = 0).$$

We now show that program (14) can be simplified by considering functions $c^i(\beta)$ which are functions of β^i only.

Lemma 1. Under A2, at the optimum $c^{*i}(\beta)$ is a function of β^i only, $i = 1, \ldots, n$.

Proof : Suppose the contrary. We show that the optimal value of program (14) can be increased, a contradiction. Denote $X^i(\beta^i) \equiv \underset{\beta^{-i}}{E} (x^i(\beta))$ and choose $c^i(\beta^i)$ such that :

$$(19) \qquad\qquad c^i(\beta^i) = \underset{\{\beta^{-i}/x^i(\beta)=1\}}{E} c^i(\beta)$$

Observe that from the linearity of the program in $(x^i(\beta))$, the optimum can be chosen so that the $(x^i(\beta))$ are zeros and ones. Then, since $\psi(0) = 0$,

(20) $\underset{\beta^{-i}}{E} \{x^i(\beta)\psi(\beta^i - C^i(\beta))\} = x^i(\beta^i) \underset{\{\beta^{-i}/x^i(\beta)=1\}}{E} \psi(\beta^i - C^i(\beta))$

Since $\Psi'' > 0$ by Jensen's inequality, the expression in (20) exceeds :

(21) $x^i(\beta^i)\psi(\underset{\{\beta^{-i}/x^i(\beta)=1\}}{E} (\beta^i - C^i(\beta))) = x^i(\beta^i)\psi(\beta^i - C^i(\beta^i))$

The maximand of (14) can therefore be replaced by the larger quantity :

(22) $E(\sum_{i=1}^{n} x^i)S - \sum_{i=1}^{n} \underset{\beta^{-i}}{E} \{\lambda U^i(\beta^i) + (1+\lambda) x^i(\beta^i)(C^i(\beta^i) + \psi(\beta^i - C^i(\beta^i)))\}$

Moreover, the constraints can be relaxed. Since $\psi'(0) = 0$ and by A2, $\psi''' \geq 0$, we have similarly :

(23) $\underset{\beta^{-i}}{E} \{x^i(\beta)\psi'(\beta^i - C^i(\beta))\} \geq x^i(\beta^i)\psi'(\beta^i - C^i(\beta^i))$

Therefore, since the objective function is decreasing in U^i, the constraints are relaxed if we replace (15) by :

(24) $\dot{U}^i(\beta^i) = - x^i(\beta^i)\psi'(\beta^i - C^i(\beta^i))$.

$\square$

For given optimal $(x^{*i}(.))$ and therefore given $X^{*i}(.)$, the optimization with respect to $(C^i(\beta^i))$ can be decomposed into n programs of the type :

(25) $\text{Max} \int_{\underline{\beta}}^{\overline{\beta}} \{-\lambda U^i(\beta^i) - (1+\lambda) x^i(\beta^i)(C^i(\beta^i) + \psi(\beta^i - C^i(\beta^i)))\} f(\beta^i) d\beta^i$

S.T.

$$(26) \qquad \dot{U}^i(\beta^i) = - X^i(\beta^i_-)\psi'(\beta^i - C^i(\beta^i))$$

$$(27) \qquad U^i(\bar{\beta}) = 0$$

Considering U^i as the state variable and C^i as the control variable, the Hamiltonian of this program is :

$$(28) \qquad H^i = [-\lambda U^i(\beta^i) - (1+\lambda)X^i(\beta^i)(C^i(\beta^i) + \psi(\beta^i - C^i(\beta^i)))]f(\beta^i)$$
$$- \mu^i(\beta^i)(-X^i(\beta^i)\psi'(\beta^i - C^i(\beta^i)))$$

The Pontryagin principle gives :

$$(29) \qquad \dot{\mu}^i(\beta^i) = \lambda \, f(\beta^i)$$

$$(30) \qquad (1+\lambda)(1-\psi'(\beta^i - C^i(\beta^i)))f(\beta^i) = \mu^i(\beta^i)\psi''(\beta^i - C^i(\beta^i))$$

$$(31) \qquad \mu^i(\underline{\beta}) = 0$$

Integrating (29) and using the transversality condition (31) we have :

$$(32) \qquad \mu^i(\beta^i) = \lambda \, F(\beta^i)$$

The optimal cost function $C^{*i}(\beta^i)$ is therefore determined by :

$$(33) \qquad (1+\lambda)(1-\psi'(\beta^i - C^{*i}(\beta^i))) = \lambda \, \frac{F(\beta^i)}{f(\beta^i)} \, \psi''(\beta^i - C^{*i}(\beta^i)) \, .$$

We then substitute the $(C^{*i}(\beta^i))$ into (14) to solve for the optimal $x^i(\beta)$.

Integrating (26), the Lagrangian can be written :

$$E_\beta \left[(\sum_{i=1}^{n} x^i(\beta))S - \lambda \sum_{i=1}^{n} \int_{\beta^i}^{\bar{\beta}} x^i(\tilde{\beta}^i) \, \psi'(\tilde{\beta}^i - C^{*i}(\tilde{\beta}^i))d\tilde{\beta}^i \right.$$

$$\left. - (1+\lambda) \sum_{i=1}^{n} x^i(\beta)\left[C^{*i}(\beta^i) + \psi(\beta^i - C^{*i}(\beta^i)) \right] \right] + E_\beta \, \gamma(\beta)(1 - \sum_{i=1}^{n} x^i(\beta))$$

$$+ E_\beta \, \nu^i(\beta) \, x^i(\beta)$$

where $\gamma(\beta)\, f(\beta_1)\ldots f(\beta_n)$ is the multiplier associated with the constraint $1 - \sum\limits_{i=1}^{n} x^i(\beta) \geq 0$ and $\nu^i(\beta)\, f(\beta_1)\ldots f(\beta_n)$ the multiplier associated with $x^i(\beta) \geq 0$.

Integrating with respect to β^{-i} the coefficient of $x^i(\beta^i)$ is then :

$$(34)\quad f(\beta^i \quad S - \lambda\, F(\beta^i)\; \Psi'(\beta^i - C^{*i}(\beta^i)) - (1+\lambda)\left[C^{*i}(\beta^i) + \Psi(\beta^i - C^{*i}(\beta^i)) \right.$$
$$f(\beta^i) - f(\beta^i)\left[\mathop{E}_{\beta^{-i}} \gamma(\beta) - \mathop{E}_{\beta^{-i}} \nu^i(\beta) \right]$$

$x^i(\beta)$ can equal one only if $\nu^i(\beta) = 0$ by complementary slackness. For i to be selected we must have (dividing (34) by $f(\beta^i)$) :

$$(35)\quad S - (1+\lambda)[C^{*i}(\beta^i) + \psi(\beta^i - C^{*i}(\beta^i))] - \lambda\frac{F(\beta^i)}{f(\beta^i)}\psi'(\beta^i - C^{*i}(\beta^i)) \geq \mathop{E}_{\beta^{-i}} \gamma(\beta) \geq 0$$

At an interior optimum in C^i, the Hamiltonian must be concave, i.e. $(1+\lambda)\psi''f + \lambda F\psi''' \geq 0$. Then, differentiating (33), we check that A1 is sufficient for C^{*i} to be non decreasing in β^i. Consequently, using A1 again, we see the left hand side of (35) is a non decreasing function of β^i.

Therefore, we must choose $x^{*i}(\beta)$ as :

$$(36)\qquad x^{*i}(\beta) = 1 \qquad\text{if}\qquad \beta^i < \min_{k\neq i} \beta^k$$

$$x^{*i}(\beta) = 0 \qquad\text{if}\qquad \beta^i > \min_{k\neq i} \beta^k$$

Hence, the $(x^{*i}(.))$ and $X^{*i}(\beta^i)$ are non increasing almost everywhere and $(C^{*i}(\beta^i))$ are non decreasing in β^i almost everywhere. The neglected second order conditions (9) are therefore satisfied. We have obtained :

Theorem 1 : Under A1, A2, an (interior) optimal auction awards the contract to the firm announcing the lowest cost parameter. The cost level (and consequently the effort level) required from the manager selected is a solution of (33). The transfer to the firm must be a solution of (8).

This result deserves some comments. The optimal auction is deterministic. The level of effort determined by equation (33)

Actually, the effort level is identical to the one we obtained with only one firm (Laffont and Tirole (1984)). Intuitively the reason is as follows. <u>Competition amounts for the best firm to a truncation of the interval $[\underline{\beta},\bar{\beta}]$ to $[\underline{\beta},\beta^*]$ where β^* is the second lowest bid</u>. But the distribution plays a role in equation (33) only through the value of $\dfrac{F(\beta^i)}{f(\beta^i)}$ which is independent of β^* (as is well known, the term $\frac{F}{f}$ comes from the trade-off between lowering the effort distorsion at β^i-weighted by the density $f(\beta^i)$- and lowering the rent of the firms which are more efficient than β^i-weighted by the c.d.f. $F(\beta^i)$). Of course, as n grows the winner's productivity parameter β^i becomes close in probability to $\underline{\beta}$, $F(\beta^i)$ becomes close to zero and the effort level is close to the optimal one. Competition solves asymptotically the moral hazard problem by solving the adverse selection problem (this extreme result clearly relies on risk neutrality).

4 - Implementation by a dominant strategy auction

a) From (10) the system of transfers of the optimal Bayesian auction is such that :

(37) $\qquad t^{*i}(\beta^i) \equiv \underset{\beta^{-i}}{E}\ t^{*i}(\beta) = U^{*i}(\beta^i) + X^{*i}(\beta^i)\psi(\beta^i - C^{*i}(\beta^i))$

Using (11) and (12) we have :

(38) $\qquad t^{*i}(\beta^i) = \displaystyle\int_{\beta^i}^{\bar{\beta}} X^{*i}(\tilde{\beta}^i)\psi'(\tilde{\beta}^i - C^{*i}(\tilde{\beta}^i))d\tilde{\beta}^i + X^{*i}(\beta^i)\psi(\beta^i - C^{*i}(\beta^i))$

We now construct a dominant strategy auction of the Vickrey type which implements the same cost function (and effort function) and also selects the most efficient firm.

Let

(39)
$$\bar{t}^i(\beta)^{4)} = \psi(\beta^i - c^{*i}(\beta^i)) + \int_{\beta^i}^{\beta^j} \psi'(\tilde{\beta}^i - c^{*i}(\tilde{\beta}^i)) d\tilde{\beta}^i$$

if $\beta^i = \min_k \beta^k$ with $\beta^j = \min_{k \neq i} \beta^k$

(40) $\bar{t}^i(\beta) = 0$ otherwise.

When firm i wins the auction its transfer is equal to the individually rational transfer <u>plus</u> the rent the firm gets when the distribution is truncated at β^j. Therefore we are back to the monopoly case, except that the truncation point β^j is random. But for any truncation point, truthtelling is optimal in the monopoly case. Therefore, truthtelling is a dominant strategy (see Appendix 3).

If $\beta^i = \bar{\beta}$, from (39), $U^i(\bar{\beta}) = 0$ for any (β^j) and therefore also in expectation. The dominant strategy auction is individually rational. Using the distribution of the second order statistics it is easy to check that $E_{\beta^{-i}} \bar{t}^i(\beta) = t^{*i}(\beta^i)$.

Therefore, the dominant strategy auction costs the same expected transfers than the optimal Bayesian auction; this is somewhat analogous to the revenue equivalence theorem.

b) Implicit in the approach with mechanisms that we have followed is the fact that if the selected firm ex-post cost differs from $c^{*i}(\beta^i)$ it incurs an infinite penalty. However, this mechanism is not robust to the introduction of a random disturbance ε^i (uncorrelated with β^i into the cost function ($c^i = \beta^i - e^i + \varepsilon^i$).

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We will now rewrite the transfer function in a form which
is closer to actual pratice and is robust to cost disturbances.

Let

(41) $\qquad s^i(\beta,C) = \bar{t}(\beta) + K(\beta^i)(C^*(\beta^i) - C)$

where $K(\beta^i) \equiv \psi'(\beta^i-C^*(\beta^i)) \in [0,1]$, noting that the optimal
cost function C^* is independent of i.

It is easy to check that (46) still induces the appropriate
ex-post effort level and truthtelling. The transfer is now decompo-
sed into a transfer $\bar{t}^i(\beta)$ computed at the time of the auction and
into a sharing of overruns (meaningfull when costs are random)
determined by the coefficient $K(\beta^i)$.

The auction selects the best firm on the market and awards
the winner an incentive contract to induce a second best level
of effort. Thus we generalize the result in Laffont and Tirole
(1984) that the optimal allocation can be implemented by offering
the firm(s) a menu of linear contracts.

In particular, the fixed transfer $\bar{t}^i$ decreases with the
winner's bid (β^i) and increases with the second bid (β^j). The
slope of the incentive scheme K decreases with the winner's bid.
Thus, the contract moves toward a fixed-price contract when the
winning bid decreases.

Lastly we can note that firm i's expected cost, if it is
chosen, is an increasing function of β^i. This implies that the
optimal auction can have the firms announce expected cost C^{ai}.
(41) can then be rewritten

(42) $\qquad s^i(C^a,C) = \tilde{t}(C^{ai},C^{aj}) + K(C^{ai})(C^{ai}-C)$

where $C^{ai} \leq C^{aj} \leq C^{ak}$ for all $k \neq i,j$.

5 - Conclusion

This paper draws a remarkably simple bridge between auction theory and incentive theory. The optimal auction truncates the uncertainty range for the successful bidder's intrinsic cost from $[\underline{\beta}, \bar{\beta}]$ to $[\underline{\beta}, \beta^j]$, where β^j is the second bidder's intrinsic cost. The principal offers the successful bidder the optimal incentive contract for a monopolist with unknown cost in $[\underline{\beta}, \beta^j]$. Previous work then implies that the successful bidder faces a linear cost reimbursement contract, the slope of which depends only on his bid, and not on the other bidders' bids.

Technically, the optimal auction can be viewed by each bidder as the optimal monopoly contract with a random upward truncation point. This trivially implies that the auction is a dominant strategy auction. Our dichotomy - the winner's fixed transfer depend on the first and the second bids, and the slope of his incentive scheme depends only on the first bid - comes from the fact that the ex-post incentive constraint is downward binding while bidding leads to an upward truncation of the conditional distribution. Lastly we should note that the optimal allocation can be implemented by a dominant strategy auction which uses information about the first and second bids[5].-

Appendix 1 : Non optimality of random schemes

In the one firm problem the optimization program of the Government is

$$\underset{C(.),U(.)}{\text{Max}} \int_{\underline{\beta}}^{\overline{\beta}} \{S-\lambda U(\beta)-(1+\lambda)[C(\beta)+\psi(\beta-C(\beta))]\}f(\beta)d\beta$$

S.T.
$$\dot{U}(\beta) = - \psi'(\beta-C(\beta))$$
$$U(\overline{\beta}) = 0.$$

Suppose that the optimal control $C^*(\beta)$ is random. We show that replacing it by $EC^*(\beta)$ improves the program a contradiction.

From Jensen's inequality and $\psi'' > 0$

$$-E\psi(\beta-C^*(\beta)) < - \psi(\beta-EC^*(\beta))$$

The maximand can be replaced by the larger quantity

$$\int_{\underline{\beta}}^{\overline{\beta}} \{S - \lambda U(\beta) - (1+\lambda)[EC^*(\beta) + \psi(\beta-EC^*(\beta))]f(\beta)\, d\beta$$

If $C^*(\beta)$ is random the constraint A1.2 must be written

$$\dot{U}(\beta) = - E\psi'(\beta-C^*(\beta))$$

From Jensen's inequality and $\psi''' \geq 0$

$$- E\psi'(\beta-C^*(\beta)) \leq - \psi'(\beta-EC^*(\beta))$$

As the maximand is decreasing in U, the value of the program is increasing if we replace $-E\psi'(\beta-\overline{C}^*(\beta))$ by $-\psi'(\beta-EC^*(\beta))$.

The same property holds in the competitive case. $\square$

Appendix 2 : Characterization of truthtelling

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To minimize technicalities we assume that the allocation is
a.e. differentiable. It can be shown that it is indeed the case.
For truthtelling to be a best Bayesian Nash strategy, β^i must
be the best answer for firm i when it assumes that the other
firms are truthful, i.e. must be the solution of

$$\underset{\widetilde{\beta}^i}{\text{Max}} \; U(\beta^i, x^i, C^i, t^i) \equiv \underset{\beta^{-i}}{E} \; \{t^i(\widetilde{\beta}^i, \beta^{-i}) - x^i(\widetilde{\beta}^i, \beta^{-i}) \psi(\beta^i - C^i(\widetilde{\beta}^i, \beta^{-i}))\}$$

The objective function satisfies the conditions CS_-, CS_+
of Guesnerie and Laffont (1984), i.e.

$$\frac{\partial}{\partial \beta^i} (\frac{\partial U / \partial x^i}{\partial U / \partial t^i}) = - \underset{\beta^{-i}}{E} \; \psi' < 0$$

$$\frac{\partial}{\partial \beta^i} (\frac{\partial U / \partial C^i}{\partial U / \partial t^i}) = \underset{\beta^{-i}}{E} \; x^i \psi'' > 0$$

Transfers must satisfy the first order condition of incenti-
ve compatibility :

$$(A2.1) \quad \frac{\partial}{\partial \beta^i} \; \underset{\beta^{-i}}{E} t^i(\beta) = - \frac{\partial}{\partial \beta^i} \; \underset{\beta^{-i}}{E} \; [x^i(\beta) \psi(\beta^i - C^i(\beta))]$$

If C^i is non decreasing in β^i a.e. and if $\underset{\beta^{-i}}{E} \; x^i(\beta)$ is non

increasing in β^i a.e., then following the proof of theorem 2 in
Guesnerie and Laffont (1984) we show that these functions $x^i(\beta)$,
$C^i(\beta)$ can be implemented by transfers solutions of A2.1.

Appendix 3 : Implementation in dominant strategy

Let us check that indeed truthtelling is a dominant stra-tegy.

Suppose first that by announcing the truth, manager i wins the auction. Since the auction is individually rational and lo-sers get nothing he cannot gain by lying and losing the auction What about an answer $\widetilde{\beta}^i \neq \beta^i$ but such that $\widetilde{\beta}^i < \min_{k \neq i} \beta^k$?

Manager i's program is :

(A3.1) $$\underset{\widetilde{\beta}^i \leq \underset{k \neq i}{\text{Min}} \beta^k}{\text{Max}} \{\overline{t}^i(\widetilde{\beta}^i, \beta^{-i}) - \psi(\beta^i - C^{*i}(\widetilde{\beta}^i))\}$$

The first order condition for an interior solution is :

(A3.2) $$\psi'(\widetilde{\beta}^i - C^{*i}(\widetilde{\beta}^i))(1 - \frac{dC^{*i}}{d\beta^i}(\widetilde{\beta}^i)) - \psi'(\widetilde{\beta}^i - C^{*i}(\widetilde{\beta}^i))$$

$$+ \psi'(\beta^i - C^{*i}(\widetilde{\beta}^i))\frac{dC^{*i}}{d\beta^i}(\widetilde{\beta}^i) = 0$$

Hence $\widetilde{\beta}^i = \beta^i$ is the only solution of the first order condition. Moreover the second order condition at $\widetilde{\beta}^i = \beta^i$ is sa-tisfied since $\frac{dC^{*i}}{d\beta^i} \geq 0$.

Finally, observe that a loser of the auction does not want to bid less than his true parameter. If he still loses the auction he gains nothing. If he gains the auction, his transfer does not compensate him for his increased effort level.

Let β^i be his true parameter and let his announcement $\widetilde{\beta}^i$ be smaller than the smallest announcement β^j.

$$U^i(\tilde{\beta}^i) = \psi(\tilde{\beta}^i - C^{*i}(\tilde{\beta}^i)) + \int_{\tilde{\beta}^i}^{\beta^j} \psi'(\tilde{\beta}^i - C^{*i}(\tilde{\beta}^i)) d\tilde{\beta}^i - \psi(\beta^i - \tilde{\beta}^i + \tilde{\beta}^i - C^{*i}(\tilde{\beta}^i))$$

$$\leq -(\beta^i - \tilde{\beta}^i)\psi'(\tilde{\beta}^i - C^{*i}(\tilde{\beta}^i)) + \int_{\tilde{\beta}^i}^{\beta^j} \psi'(\tilde{\beta}^i - C^{*i}(\tilde{\beta}^i)) d\tilde{\beta}^i$$

$$\leq -(\beta^i - \tilde{\beta}^i)\psi'(\tilde{\beta}^i - C^{*i}(\tilde{\beta}^i)) + (\beta^j - \tilde{\beta}^i)\psi'(\tilde{\beta}^i - C^{*i}(\tilde{\beta}^i))$$

as (from (33)), $\dfrac{de^{*i}}{d\beta^i} = 1 - \dfrac{dC^{*i}}{d\beta^i} \leq 0$.

Since $\beta^i \geq \beta^j$, $U^i(\tilde{\beta}^i) \leq 0$.

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Footnotes

1) In Laffont and Tirole (1985) we study dynamic contracting
    but with a single firm.

2) If A1 is not satisfied the methods described in Guesnerie
    and Laffont (1984) must be used.

3) From the revelation principle, we know that there is no
    loss of generality in restricting the analysis to these
    direct mechanisms (see e.g. Riley and Samuelson (1981)).

4) Note that as n is large β^j is close to β^i in probability
    and the transfer converges to the disutility of the "second
    lowest bidder".

5) A precursor to our paper is Mc Afee - Mc Millan (1984) (see
    also Fishe - Mc Afee (1983)). Their paper looks at the trade-
    off between giving the agent incentives for cost reduction,
    stimulating bidding competition and sharing risk. They res-
    trict the contract to be linear in observed cost and in the
    successful bidder's bid ("first bid auction"). Our paper
    shows that, under risk neutrality, the optimal contract can
    indeed be taken to be linear in observed costs. However the
    coefficient of risk sharing should decrease with the success-
    ful bidder's bid.

    Riordan and Sappington (1985) consider a framework com-
    plementary to ours. Bidders submit bids and then learn their
    true marginal cost. Each firm's private information at the
    bidding stage is a signal correlated with final cost. Thus
    they are interested in repeated adverse selection, as opposed
    to ex-ante adverse selection and ex-post moral hazard. In
    the independent signals case they consider two classes of

auctions. One is a first price auction, in which the successful bidder's allocation depends only on his bid, not on the other bidder's bid. The other is a second price auction, in which the winner gets the terms corresponding to the second bid, independently of the value of the first bid. Riordan and Sappington show that the revenue equivalence theorem does not hold.

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